# Power Control For Multiuser Space-Time CDMA

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***Abstract*— This paper discusses power control for the uplink of an asynchronous DS-CDMA system using a space-time MMSE receiver with successive interference cancellation (SIC). An iterative power control algorithm is shown to converge to the optimum distribution, even in the presence of signal cancellation errors. The optimum power control algorithm requires significant feedback between the receiver and transmitter. A low-rate, fixed-step, binary (or ternary) feedback, power control algorithm is shown to converge to close to the optimum distribution. Simulations of a multi-cellular CDMA system demonstrate that a space-time MMSE-SIC receiver with a fixed-step, binary feedback power control provides significant capacity improvements over a conventional single user matched-filter receiver.**

## I. Introduction

This paper discusses power control for the uplink (user terminal to base station) of an asynchronous, direct sequence CDMA system with a space-time, multiuser receiver at the base station. In space-time processing, signals from multiple antennas at the receiver are operated on in both the spatial and temporal domain to combat multipath and multiple access interference and thus increase the spectral efficiency of the system [2]. However, along with receiver signal processing to combat multiple access interference, there is a need for transmit optimization in the form of power control in order to ensure that each user meets its target data rate and QOS (Quality Of Service) requirements.

There has been substantial work on power control for CDMA systems. [10] proposed a distributed power control scheme for a single user matched-filter receiver and proved convergence to a feasible solution whenever such a feasible solution exists. [8] proposed a general framework for studying uplink power control in cellular wireless systems. [14], [15] addressed the issue of power allocation for a linear, single antenna, MMSE multiuser receiver for a synchronous CDMA system with no multipath. The filter coefficients that minimize the mean square error depend on the power allocation, and the optimal power allocation depend on the receiver filter coefficients. Because of this recursive relationship between the filter coefficients and the power allocation, there is no known analytical expression relating the two quantities. [14], [15] proposed an iterative scheme that converged to the optimal power allocation and filter coefficients. [18] discussed jointly optimal power control and beamforming for a CDMA system with multiple receive antennas and no multipath. A linear, minimum variance distortionless response (MVDR) beamformer is used for the spatial processing while a conventional matched-filter receiver is used for the temporal processing. [17] discussed jointly optimal power allocation and linear, temporal-spatial processing for a synchronous CDMA with multiple receive antennas and no multipath. Both [18] and [17] proposed iterative algorithms that converge to the optimal power distribution.

[14], [15], [18], [17] discussed power control for a linear MMSE receiver. This paper addresses the issue of power allocation for a non-linear MMSE receiver with succeessive interference cancellation (SIC). The effect of signal cancellation errors is included in the analysis. Although there is no analytical expression for the optimum power distribution, an iterative power control algorithm is shown to converge to the optimum power allocation.

The wireless channel is quite dynamic, and the power allocation needs to be updated as users enter, move within and leave the system. The optimum, iterative power control algorithm requires a fairly high level of feedback between the receiver and each transmitter. A lower feedback power control algorithm is preferable. [5] and [6] showed that for a conventional matched-filter receiver, a ternary, fixed-step iterative scheme (Up/Down/No Change) converges to close to the optimum power distribution. [16] proved convergence of the binary, fixed-step power control algorithm for a conventional matched-filter receiver. This paper proves that both the binary and ternary feedback power control algorithms converge to close to the optimum distribution with any multiuser space-time receiver satisfying some general properties. The MMSE-SIC receiver is analyzed in detail and shown to satisfy these properties. Simulations are used to determine the system capacity of a multicellular CDMA system with an MMSE-SIC receiver. Simulations demonstrate that even with fixed-step, binary feedback power control, the system capacity of an MMSE-SIC receiver is significantly higher than that of a conventional single user matched-filter receiver . Thus, power control for space-time MMSE receivers with SIC can be designed with no additional feedback relative to that of a conventional single user matched-filter reciever.

This paper is organized as follows. Section II describes the system model used in this paper. Section III discusses power control for the space-time MMSE receiver with SIC. Section IV discusses low feedback, iterative power control for the MMSE-SIC receiver. Section V presents some simulation results.

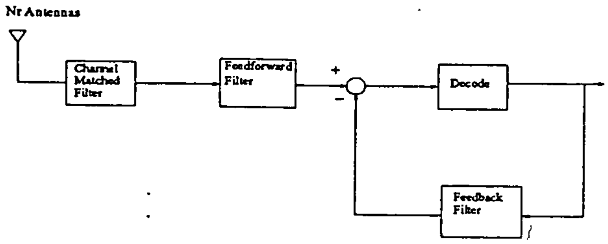


Fig. 1. General model of the MMSE receiver with ideal SIC

## II. System Model

The system considered in this paper is a fully asynchronous system in which after despreading with the desired signature sequence, other user's signals are modeled as pseudorandom, Bernoulli ($\pm 1$) sequences. Only user powers are considered in the analysis and actual transmitted bits and other subtleties affecting power control are ignored.

Each transmit signal goes through a multipath channel, $\mathbf{h}_k(t) \in \mathcal{C}^{N_r}$, modeled as:

$$\mathbf{h}_k(t) = \sum_{l=0}^{L-1} \mathbf{a}_{k,l}\delta(t - \tau_{k,l}) \tag{1}$$

where $N_r$ is the number of antennas at the receiver, $L$ is the number of resolvable paths, $\mathbf{a}_{k,l} \in \mathcal{C}^{N_r}$ is the spatial channel response for the $l^{th}$ path, and $\tau_{k,l}$ is the path delay. Asynchronous transmission by the users is represented by different path delays, $\tau_{k,l}$.

Let $\mathbf{p}(n) \in \mathcal{R}^K$ be the received power vector and $\mathbf{i}(\mathbf{p}(n)) \in \mathcal{R}^K$ as the corresponding received interference and noise power vector for the $K$ users in the system at the $n^{th}$ power control iteration. Then the SINR (signal to interference and noise power ratio), $\hat{\gamma}_k(n) = \frac{p_k(n)}{i_k(\mathbf{p}(n))}$, where $p_k(n)$ is the user power and $i_k(\mathbf{p}(n))$ is the interference and noise power for the $k^{th}$ user. The received power vector, $\mathbf{p}$, is considered *feasible*, if $\hat{\gamma}_k \geq \gamma_k, \forall k$, where $\gamma_k$ is the desired SINR for the $k^{th}$ user. Thus, for a feasible power vector,

$$\mathbf{p} \geq \Lambda \mathbf{i}(\mathbf{p}) \tag{2}$$

where $\Lambda$ is a diagonal matrix of target SINR's, i.e.

$$\Lambda_{ij} = \begin{cases} 0, & i \neq j \\ \gamma_i, & i = j \end{cases} \tag{3}$$

Likewise, an interference vector, $\mathbf{i}(\mathbf{p})$, is considered feasible if there exists a feasible power vector $\mathbf{p}$.

## III. Space-Time MMSE receiver with SIC

The optimal MMSE receiver with ideal SIC is discussed in detail in [11]. A very high level description of the optimal receiver as described in [11] is shown in Figure 1. However, this model of the MMSE-SIC receiver does not account for signal cancellation errors. Moreover, the optimal power allocation is more easily determined using the alternate representation of the MMSE-SIC receiver as shown in Figure 2. In this representation, the receiver delays the received signal to align the different multipath components and despreads the delayed signal by the user's signature sequence. To suppress the residual other user and multipath interference, each receiver uses a set of $L$, $N_r$ dimensional spatial filters, $\mathbf{b}_{kl}$, one for each multipath. The decoded signal is re-encoded and cancelled from the received signal prior to decoding subsequent users. The spatial filters are normalized such that

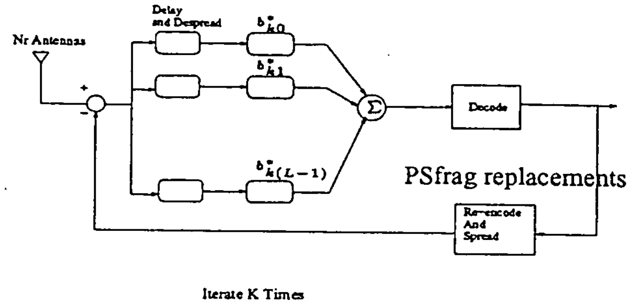


Fig. 2. Alternative representation of MMSE-SIC receiver

$$\|\sum_{l=0}^{L-1} \mathbf{b}_{kl}^* \mathbf{a}_{kl}\|^2 = 1 \tag{4}$$

With these assumptions,

$$\begin{aligned} i_k(\mathbf{p}(n)) = \;& \min_{\mathbf{b}_{kl}} (\textstyle\sum_{j=0}^{k-1} \varepsilon_{kj} F_{kj} p_j(n) + & (5)\\ & \textstyle\sum_{j=k}^{K-1} F_{kj} p_j(n) + \sigma_k^2) & (6) \end{aligned}$$

where

$$F_{kj} = \sum_{l=0}^{L-1} \sum_{u=0,(k,l)\neq(j,u)}^{L-1} \|\mathbf{b}_{kl}^* \mathbf{a}_{ju}\|^2 \tag{7}$$

$$\varepsilon_{kj} \stackrel{\text{def}}{=} \frac{\sum_{l=0}^{L-1}\sum_{u=0}^{L-1} \|\mathbf{b}_{kl}^* \mathbf{e}_{ju}\|^2}{\sum_{l=0}^{L-1}\sum_{u=0}^{L-1} \|\mathbf{b}_{kl}^* \mathbf{a}_{ju}\|^2} \tag{8}$$

$$\sigma_k^2 = \frac{N_0}{2} \sum_{l=0}^{L-1} \|\mathbf{b}_{kl}\|^2 \tag{9}$$

$\mathbf{e}_{ju} \in \mathcal{C}^{N_r}$ is the spatial error in cancelling the $u^{th}$ path of the $j^{th}$ user (i.e. $\mathbf{e}_{ju} \stackrel{\text{def}}{=} \mathbf{a}_{ju} - \hat{\mathbf{a}}_{ju}$), and $\frac{N_0}{2}$ is the two-sided power spectral density of the additive, white noise. The optimum filter coefficients can be calculated as [18], [2]

$$\mathbf{b}_{kl} = \Gamma_{kl} \frac{\Phi_{kl}^{-1} \mathbf{a}_{kl}}{\mathbf{a}_{kl}^* \Phi_{kl}^{-1} \mathbf{a}_{kl}} \tag{10}$$

where

$$\begin{aligned}\Phi_{kl} &= \sum_{j=0}^{j<k}\sum_{u=0}^{L-1} p_j \mathbf{e}_{j,u}\mathbf{e}_{j,u}^* + \sum_{u=0,u\neq l}^{L-1} p_k \mathbf{a}_{k,u}\mathbf{a}_{k,u}^* \\ &+ \sum_{j=k+1}^{K-1}\sum_{u=0}^{L-1} p_j \mathbf{a}_{j,u}\mathbf{a}_{j,u}^* + \frac{N_0}{2}\mathbf{I} \\ \Gamma_{kl} &= p_k \mathbf{a}_{kl}^* \Phi_{kl}^{-1}\mathbf{a}_{kl}\end{aligned}$$

If $\varepsilon_{k,j} = 1, \forall k, j$ (no cancellation), then this receiver is equivalent to the linear MMSE receiver discussed in [17], [2]. If there is no multipath, and $\varepsilon_{kj} = 0, \forall k, j$, (perfect cancellation) then [7] proved that this receiver is equivalent to the MMSE-SIC receiver described in [11] and shown in Figure 1.

The optimal filter coefficients depend on the power allocation. Let $\mathbf{i}(\mathbf{p})$ be the resulting interference vector given the power allocation vector, $\mathbf{p}$. The following theorem proves the existence of an optimal power allocation, and provides an iterative technique for determining the optimal power vector.

*Theorem 1:* If the interference vector for the MMSE-SIC receiver is feasible, then there exists a unique, optimal solution $\mathbf{p}^* = \Lambda\mathbf{i}(\mathbf{p}^*)$. For any other feasible power vector, $\mathbf{p}$, the received power for each user is greater than that of the optimal solution, i.e. $\mathbf{p} \geq \mathbf{p}^*$. The iterative algorithm $\mathbf{p}(n+1) = \Lambda\mathbf{i}(\mathbf{p}(n))$ converges to the optimal power distribution.

*Proof*: The proof requires understanding of *Standard* interference vectors as defined in [8]. In [8], Yates defined a *standard* interference vector, $\mathbf{i}(\mathbf{p})$, as one that satisfies the following properties:

1) *Positivity:* $\mathbf{i}(\mathbf{p}) > 0$
2) *Monotonicity:* If $\mathbf{p} \geq \tilde{\mathbf{p}}$, then $\mathbf{i}(\mathbf{p}) \geq \mathbf{i}(\tilde{\mathbf{p}})$
3) *Scalability:* $\forall \alpha > 1, \alpha\mathbf{i}(\mathbf{p}) > \mathbf{i}(\alpha\tilde{\mathbf{p}})$

The following Lemma from [8] describes the conditions for the existence of the optimal power vector.

*Lemma 1:* If a *standard* interference vector is feasible, there exists a unique, optimal solution $\mathbf{p}^* = \Lambda\mathbf{i}(\mathbf{p}^*)$. For any other feasible power vector, $\mathbf{p}$, the received power for each user is greater than that of the optimal solution, i.e. $\mathbf{p} \geq \mathbf{p}^*$. The iterative algorithm $\mathbf{p}(n+1) = \Lambda\mathbf{i}(\mathbf{p}(n))$ converges to the optimal power distribution.

Hence, Theorem 1 holds true if it can be proved that the MMSE-SIC receiver results in a *standard* interference vector. Let $\mathbf{i}(\mathbf{p}, \mathbf{b}_{kl})$ be the interference vector as a function of the power vector $\mathbf{p}$, and the $KL$ spatial filters $\mathbf{b}_{kl}, k = 0, \ldots, K-1, l = 0, \ldots, L-1$. It is relatively straightforward to show that given a set of spatial filters, $\mathbf{i}(\mathbf{p}, \mathbf{b}_{kl})$ satisfies the properties of *Positivity, Monotonocity* and *Scalability*, and is hence a *standard* interference vector. But for a space-time MMSE-SIC receiver,

$$\mathbf{i}(\mathbf{p}) = \min_{\mathbf{b}_{kl}}(\mathbf{i}(\mathbf{p}, \mathbf{b}_{kl})) \tag{11}$$

The following Lemma from [8] proves that $\mathbf{i}(\mathbf{p})$ for an MMSE-SIC receiver is also a *Standard* interference vector.

*Lemma 2:* If $\mathbf{i}^{(a)}(\mathbf{p})$ and $\mathbf{i}^{(b)}(\mathbf{p})$ are two *standard* interference vectors, then $\mathbf{i}(\mathbf{p}) = \min(\mathbf{i}^{(a)}(\mathbf{p}), \mathbf{i}^{(b)}(\mathbf{p}))$ is also a *standard* interference vector.

## IV. Fixed-Step Iterative Power Control

Theorem 1 proves that the iterative algorithm $\mathbf{p}(n+1) = \Lambda\mathbf{i}(\mathbf{p}(n))$ converges to the optimal power distribution for an MMSE receiver with SIC. However, this iteration requires significant feedback from the receiver to the transmitter, especially for fast, time-varying channels. For practical implementations, it is necessary to have a low feedback iterative power control scheme that converges to close to the optimum solution. This paper proves that fixed-step iterative schemes with either binary feedback (Up/Down) or ternary feedback (Up/Down/No Change) converge to close to the optimal distribution for all *standard* interference vectors. Consequently, this paper proves that power control for the MMSE receiver with SIC does not require additional feedback relative to a single user matched-filter receiver.

### A. Fixed-step with Binary Feedback

Consider the fixed-step binary feedback iterative algorithm:

$$p_k(n+1) = \begin{cases} \delta p_k(n), & \text{if } p_k(n) \leq \gamma_k i_k(\mathbf{p}(n)) \\ \delta^{-1} p_k(n), & \text{if } p_k(n) > \gamma_k i_k(\mathbf{p}(n)) \end{cases} \tag{12}$$

where $\delta > 1$ is the power control step-size. [16] proved that if the interference vector is feasible, then for a matched-filter receiver the iterative algorithm in (12) converges to a power distribution such that the resulting SINR is within $\delta^2$ of the target SINR for every user. This section now proves convergence of (12) for any feasible, *standard*, interference vector.

*Theorem 2:* If a *standard* interference vector is feasible with an SINR margin of $\delta$, the fixed-step binary feedback algorithm in (12) converges monotonically to a power distribution at time $n_0$ such that

$$\delta^{-2}\gamma_k \leq \hat{\gamma}_k(n) \leq \delta^2\gamma_k, \forall n \geq n_0 \tag{13}$$

*Proof*: The proof relies on the following propositions. The proofs of the propositions are presented in the appendix.

*Proposition 1*: While $\hat{\gamma}_k(n) > \gamma_k$, $\hat{\gamma}_k(n)$ decreases monotonically every power control update until $\gamma_k \geq \hat{\gamma}_k(n) > \delta^{-2}\gamma_k$.

*Proposition 2*: While $\hat{\gamma}_k(n) \leq \gamma_k$, then $\hat{\gamma}_k(n)$ increases monotonically every power control update.

*Proposition 3*: If the interference vector is feasible with an SINR margin of $\delta$, and $\hat{\gamma}_k(n) \leq \gamma_k$, then $\hat{\gamma}_k(n)$ increases monotonically every power control update until

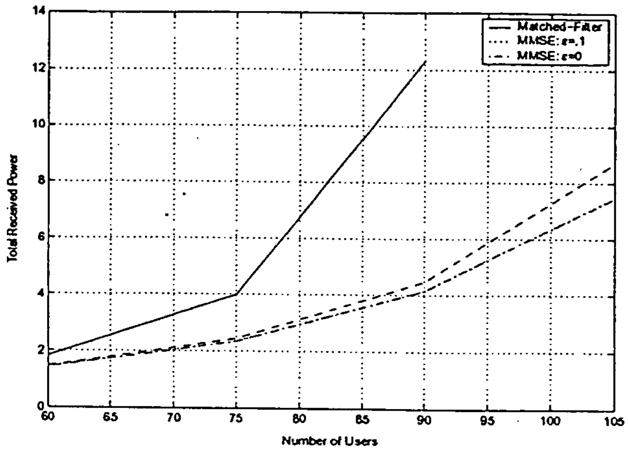


Fig. 3. Total Received Power using optimum power control

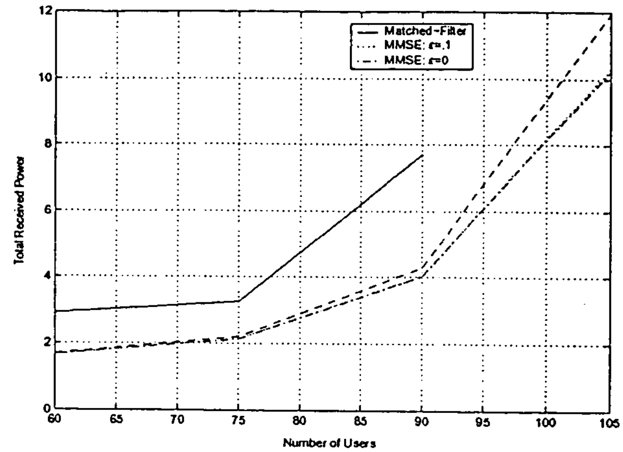


Fig. 4. Total Received Power with fixed-step binary feedback power control

$\gamma_k < \hat{\gamma}_k(n) < \delta^2\gamma_k$.

*Proposition 4:* If $\delta^{-2}\gamma_k < \hat{\gamma}_k(n_0) < \delta^2\gamma_k$, then $\delta^{-2}\gamma_k < \hat{\gamma}_k(n) < \delta^2\gamma_k, \forall n \geq n_0$.

Thus, irrespective of the initial condition, each user's SINR would eventually be bounded within $[\delta^{-2}, \delta^2]$ of the target SINR. Propositions 1, 2 and 4 do not require the interference vector to be feasible. However, Proposition 3 requires feasibility with an SINR margin of $\delta$. This basically means that if the target SINR matrix is $\Lambda$, then $\delta\Lambda$ should also be a feasible, target SINR matrix.

### B. Fixed-step with Ternary Feedback

[5] proposed the following fixed-step ternary feedback algorithm:

$$p_k(n+1) = \begin{cases} \delta p_k(n), & \text{if } p_k(n) < \delta^{-1}\gamma_k i_k(\mathbf{p}(n)) \\ \delta^{-1} p_k(n), & \text{if } p_k(n) > \delta\gamma_k i_k(\mathbf{p}(n)) \\ p_k(n), & \text{Otherwise} \end{cases} \tag{14}$$

[5] proved convergence of this algorithm for a conventional matched-filter reciever. This paper presents the following Theorem without proof. The proof is similar to that of Theorem 2.

*Theorem 3:* If a *standard* interference vector is feasible, then the fixed-step ternary feedback algorithm in (14) converges monotonically to a power distribution at time $n_0$ such that

$$\delta^{-2}\gamma_k \leq \hat{\gamma}_k(n) \leq \delta^2\gamma_k, \forall n \geq n_0 \tag{15}$$

## V. SIMULATIONS

For the simulations, the cells are laid out on a hexagonal grid. $K$ users are uniformly distributed within the cell, and an identical distribution is assumed in the adjacent cells. Only the adjacent cells are used for calculating inter-cell interference. The path gain for each user is proportional to $r^{-m}10^{-\zeta/10}$, where $r$ is the distance of the user to the base station, $m$ is the path loss exponent, and $\zeta$ is a log normal random variable that models the path loss due to shadowing [12]. The path loss exponent is set to 4, as is typical in urban deployments [12]. Each user is assumed to have independent log-normal shadowing to each of the cells with a standard deviation of 8 dB [12]. Handoff is simulated by assuming that each user is power controlled by the base station with the largest path gain. Only the adjacent cells are used for handoff calculations.

There is no multipath. There are $N_r = 4$ antennas at the base station receiver and one antenna at each mobile transmitter. The $N_r \times K$ channel matrix, $\mathbf{H}$, is modeled as a random matrix with each element being an i.i.d complex, circularly symmetric, Gaussian random variable with unit variance. This models rayleigh fading effects due to local scattering at the base station antennas. For successive interference cancellation, users are decoded in decreasing order of average path gains (including log normal shadowing but not including Rayleigh fading). The system bandwidth is 1.2288 MHz, and each user has a target data rate of 4.8 kbps[1] with an $E_b/N_0$ requirement of 5 dB. Figure 3 shows the total received power with optimum power control. It shows that substantial benefits in system performance are achieved with an MMSE-SIC receiver even if there is some cancellation error. Figure 4 shows the total received power with binary feedback iterative power control. It shows that even with signal cancellation errors and binary feedback, the MMSE-SIC receiver with power control performs significantly better than the matched-filter receiver. Note that in both cases, the matched-filter receiver cannot even support more than 90 users.

## VI. CONCLUSION

This paper has proved the existence of a unique, optimal power allocation for an MMSE receiver with successive inter-

[1] typical average rate for voice communications

ference cancellation. An iterative algorithm is proved to converge to the optimal distribution. The effect of signal cancellation errors is included in the analysis. A low feedback, fixed-step iterative algorithm with binary or ternary feedback is shown to converge to close to the optimum distribution. Simulations demonstrate significant performance improvements with an MMSE-SIC receiver with respect to a single-user matched-filter receiver. There is minimal performance degradation due to binary feedback iterative power control and moderate cancellation errors. Thus, this paper demonstrates that power control for an MMSE-SIC receiver is robust in the presence of signal cancellation errors and does not require additional feedback relative to a single user matched-filter receiver.

## APPENDIX

### A. *Binary Feedback Power Control*

*Proposition 1*: While $\hat{\gamma}_k(n) > \gamma_k$, $\hat{\gamma}_k(n)$ decreases monotonically every power control update until $\gamma_k \geq \hat{\gamma}_k(n) > \delta^{-2}\gamma_k$.

*Proof*: Define $s_k(n) \stackrel{\text{def}}{=} \frac{\hat{\gamma}_k(n+1)}{\hat{\gamma}_k(n)}$ as the relative increase in SINR for the $k^{th}$ user at the $n^{th}$ power control iteration. By the power control algorithm of (12), under the conditions of the proposition, $p_k(n)$ would be decreased every power control update. Hence

$$\begin{aligned} s_k(n) &= \frac{p_k(n+1)i_k(\mathbf{p}(n))}{p_k(n)i_k(\mathbf{p}(n+1))} && (16)\\ &= \frac{\delta^{-1}i_k(\mathbf{p}(n))}{i_k(\mathbf{p}(n+1))} && (17)\end{aligned}$$

The maximum increase in SINR on any power control iteration, $\max(s_k(n))$, can be calculated as

$$\begin{aligned} \max(s_k(n)) &= \max(\frac{\delta^{-1}i_k(\mathbf{p}(n))}{i_k(\mathbf{p}(n+1))} && (18)\\ &= \delta^{-1}\frac{i_k(\mathbf{p}(n))}{\min(i_k(\mathbf{p}(n+1)))} && (19)\end{aligned}$$

Because of the *Monotonicity* property of the *standard* interference vector, $\min(i_k(\mathbf{p}(n+1))) = i_k(\min(\mathbf{p}(n+1)))$. But $\min(\mathbf{p}(n+1)) \geq \delta^{-1}\mathbf{p}(n)$. Hence,

$$\begin{aligned} \max(s_k(n)) &= \delta^{-1}\frac{i_k(\mathbf{p}(n))}{i_k(\min(\mathbf{p}(n+1)))} && (20)\\ &\leq \delta^{-1}\frac{i_k(\mathbf{p}(n))}{i_k(\delta^{-1}\mathbf{p}(n))} && (21)\end{aligned}$$

By the *Scalability* property of the *standard* interference vector, $i_k(\delta^{-1}\mathbf{p}(n)) > \delta^{-1}i_k(\mathbf{p}(n))$. Hence

$$\begin{aligned} \max(s_k(n)) &< \delta^{-1}\frac{i_k(\mathbf{p}(n))}{\delta^{-1}i_k(\mathbf{p}(n))} && (22)\\ &< 1 && (23)\end{aligned}$$

Since, the maximum increase in SINR is less than 1, the SINR decreases monotonically.

While $\hat{\gamma}_k(n) > \gamma_k$, $p_k(n)$ would be decreased by $\delta$ on every power control update. Eventually, $p_k(n) \to 0$. However, by the *Positivity* of the interference vector, $i_k(\mathbf{p}(n)) > 0$. Hence, it is not possible for $p_k(n) \to 0$, and still satisfy the constraint that $\hat{\gamma}_k(n) > \gamma_k$. Consequently, at some time $n_0$, $\gamma_k(\hat{n_0}) \leq \gamma_k$.

Now, the maximum decrease in SINR is $\min(s_k(n))$. This can be calculated as:

$$\begin{aligned} \min(s_k(n)) &= \min(\frac{\delta^{-1}i_k(\mathbf{p}(n))}{i_k(\mathbf{p}(n+1))} && (24)\\ &= \delta^{-1}\frac{i_k(\mathbf{p}(n))}{\max(i_k(\mathbf{p}(n+1)))} && (25)\\ &= \delta^{-1}\frac{i_k(\mathbf{p}(n))}{i_k(\max(\mathbf{p}(n+1)))} && (26)\\ &\geq \delta^{-1}\frac{i_k(\mathbf{p}(n))}{i_k(\delta\mathbf{p}(n))} && (27)\\ &> \delta^{-1}\frac{i_k(\mathbf{p}(n))}{\delta i_k(\mathbf{p}(n))} && (28)\\ &> \delta^{-2} && (29)\end{aligned}$$

Hence, $\gamma_k(\hat{n_0}) > \delta^{-2}\gamma_k$

*Proposition 2*: While $\hat{\gamma}_k(n) \leq \gamma_k$, then $\hat{\gamma}_k(n)$ increases monotonically every power control update.

*Proof*: If $\hat{\gamma}_k(n) \leq \gamma_k$, then by the power control algorithm of (12), $p_k(n+1) = \delta p_k(n)$. Hence, the minimum increase in SINR, $\min(s_k(n))$, is calculated as:

$$\begin{aligned} \min(s_k(n)) &= \min(\frac{\delta i_k(\mathbf{p}(n))}{i_k(\mathbf{p}(n+1))} && (30)\\ &= \delta\frac{i_k(\mathbf{p}(n))}{\max(i_k(\mathbf{p}(n+1)))} && (31)\\ &= \delta\frac{i_k(\mathbf{p}(n))}{i_k(\max(\mathbf{p}(n+1)))} && (32)\\ &\geq \delta\frac{i_k(\mathbf{p}(n))}{i_k(\delta\mathbf{p}(n))} && (33)\\ &> \delta\frac{i_k(\mathbf{p}(n))}{\delta i_k(\mathbf{p}(n))} && (34)\\ &> 1 && (35)\end{aligned}$$

Since, the minimum increase in SINR is greater than, this implies that the SINR increases monotonically.

*Proposition 3*: If the interference vector is feasible with an SINR margin of $\delta$, and $\hat{\gamma}_k(n) \leq \gamma_k$, then $\hat{\gamma}_k(n)$ increases monotonically every power control update until $\gamma_k < \hat{\gamma}_k(n) < \delta^2\gamma_k$.

*Proof*: Suppose the Proposition were not true and $\hat{\gamma}_k(n) \leq \gamma_k, \forall n$. Then by the power control algorithm of (12), $p_k(n) \to \infty$. We now show that these two conditions are inconsistent.

Let $\mathbf{p}^*$ be the optimal, feasible power vector with an SINR margin of $\delta$. Then $\alpha\mathbf{p}^*, \alpha \geq 1$ is also feasible with the same SINR margin because of the *Scalability* property of the standard interference vector [8]

$$\alpha\mathbf{p}^* = \delta\alpha\Lambda\mathbf{i}(\mathbf{p}^*) \quad (36)$$

$$\geq \delta\Lambda\mathbf{i}(\alpha\mathbf{p}^*) \quad (37)$$

Select $\alpha \geq 1$, such that $\mathbf{p}(n) \leq \alpha\mathbf{p}^*$. Then, by the *Monotonicity* property of the standard interference vector, $\mathbf{i}(\alpha\mathbf{p}^*) \geq \mathbf{i}(\mathbf{p}(n))$. Hence,

$$\alpha\mathbf{p}^* \geq \delta\Lambda\mathbf{i}(\alpha\mathbf{p}^*) \quad (38)$$

$$\geq \delta\Lambda\mathbf{i}(\mathbf{p}(n)) \quad (39)$$

If $\hat{\gamma}_k(n) \leq \gamma_k$, then $p_k(n) \leq \gamma_k i_k(\mathbf{p}(n))$ and $p_k(n+1) = \delta p_k(n)$. Hence

$$\alpha p_k^* \geq \delta p_k(n) \quad (40)$$

$$\geq p_k(n+1) \quad (41)$$

If $\hat{\gamma}_k(n) > \gamma_k$, then $p_k(n+1) = \delta^{-1}p_k(n)$ and quite obviously $\alpha p_k^* > p_k(n+1)$. Thus, under all conditions, $\alpha\mathbf{p}^* \geq \mathbf{p}(n+1)$. Hence, $\alpha\mathbf{p}^*$ serves as an upper bound on $\mathbf{p}(n)$, thus contradicting the assumption that $p_k(n) \rightarrow \infty$. Hence, $\exists n_0 : \hat{\gamma}_k(n_0) > \gamma_k$.

The maximum increase in SINR, $\max(s_k(n))$, is calculated as:

$$\max(s_k(n)) = \max\left(\frac{\delta i_k(\mathbf{p}(n))}{i_k(\mathbf{p}(n+1))}\right) \quad (42)$$

$$= \delta\frac{i_k(\mathbf{p}(n))}{\min(i_k(\mathbf{p}(n+1)))} \quad (43)$$

$$= \delta\frac{i_k(\mathbf{p}(n))}{i_k(\min(\mathbf{p}(n+1)))} \quad (44)$$

$$\leq \delta\frac{i_k(\mathbf{p}(n))}{i_k(\delta^{-1}\mathbf{p}(n))} \quad (45)$$

$$< \delta\frac{i_k(\mathbf{p}(n))}{\delta^{-1}i_k(\mathbf{p}(n))} \quad (46)$$

$$< \delta^2 \quad (47)$$

Consequently, $\hat{\gamma}_k(n_0) < \delta^2\gamma_k$.

*Proposition 4*: If $\delta^{-2}\gamma_k < \hat{\gamma}_k(n_0) < \delta^2\gamma_k$, then $\delta^{-2}\gamma_k < \hat{\gamma}_k(n) < \delta^2\gamma_k, \forall n \geq n_0$.

*Proof*: Consider the following two cases:

- $\delta^2\gamma_k > \hat{\gamma}_k(n) > \gamma_k$: Then, from, Proposition 1, $\max(s_k(n)) < 1$, and $\min(s_k(n)) > \delta^{-2}$. Hence, $\delta^2\gamma_k > \hat{\gamma}_k(n+1) > \delta^{-2}\gamma_k$.
- $\gamma_k \geq \hat{\gamma}_k(n) > \delta^{-2}\gamma_k$: Then, from, Proposition 2, $\min(s_k(n)) > 1$, and from Proposition 3, $\max(s_k(n)) < \delta^2$. Hence, $\delta^2\gamma_k > \hat{\gamma}_k(n+1) > \delta^{-2}\gamma_k$.

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